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STOCHASTIC PREDICTION OF MONTHLY INFLATION RATES THROUGH KALMAN FILTERING

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ABSTRACT

Inflation measure is an important indicator of the state of an economy and the desire to determine it ahead of "time" cannot be overemphasised. This paper presents a step-by-step algorithm to predict the would-be monthly inflation rate of the Nigerian economy, using Kalman Filtering Predictor (KFP). The ordinary structural model for a time series (structTS) is highlighted to "fairly" compete against our proposed KFP. The structTS is a powerful "competitor", it is in recommended R package "stats" and used for fitting basic structural models to "univariate" time series. It is quite reliable and fast, and is used as a benchmark in some comparisons of filtering techniques, it is indeed the "predictor" to "beat", yet our proposed KFP has more to "offer". The pertinent statistics and pictorial representation of the results obtained, through both techniques, is highlighted for any "incorruptible" judge's perusal. All of these are contained in the couple of illustrative examples that exhibit the steps involved in the proposed algorithm, using a hypothetical monthly inflation rate and the monthly inflation rates data (January, 2011 to June, 2014) of the Nigerian economy.

Keywords: Monthly Inflation Rates (MIR), Structural model for Time Series (structTS), Probabil Spaces, Kalman Filtering Predictor (KFP) and R Package.

INTRODUCTION

There are many causes of inflation in a developing economy like Nigeria. The calculation of monthly inflation rates (MIR) and their predictions are by no means easy, yet there is the continual need of being able to predict them for effective monitoring and control of an ailing economy. The decision making bodies of Businesses in Nigeria often require the regular predictions of the MIR for effective control and avoidance of economic pitfalls. The problem pertaining to effective prediction of inflation rates of economies around the world has "grasped" the attention of many experts since the last three decades. The notable ones are (Engle, 1982; Shah *et al*, 2014; Awogbemi and Ajao, 2011; Maku and Adelowokan, 2013). In Nigeria, the inflation rate is not stable; its wanton behaviour further emphasizes the need for it to be predicted for effective decision making and routine running of businesses. For instance, Nigeria's inflation rate moved from 13% in the second quarter to 13.7% in the third quarter of 2010 (CBN, 2010). Numerous applications, such as in finance and engineering, also require either the estimation, prediction or modelling of noisy time series. Prediction is concerned with using all available data to approximate a future value of the series. Inflation is marked by an increase in the general level of prices or a decrease in the value of money. It is highly affected by interrelated economic, social, political and even psychological factors. These factors interact with each other in a complicated manner. Forecasting inflation is one of the more important, but difficult, exercises in macroeconomics. Many different approaches have been suggested. Such approaches include the works of Ang et al (2007), Atkeson and Ohanian (2001), Groen *et al* (2010), Stock and Watson (1999) and Stock and Watson (2008). Maku and Adelowokan (2013) examined the determinants of inflation rate, amidst macroeconomic fluctuations in Nigeria between a decade after independence 1970 and 2011.

Their study employed a partial adjustment of autoregressive model and it indicated that fiscal deficit and interest rate exert decelerating pressure on dynamics of inflation rate in Nigeria. While, other macroeconomic indicators such as real output growth rate, broad money supply growth rate, and previous level of inflation rate further exert increasing pressure on inflation rate in Nigeria. They concluded that there is significant adjustment process of the dynamics of inflation rate in Nigeria, while real output growth rate and fiscal deficit are significant determinants of inflation rate in Nigeria.
In Statistics and Economics, a filter is simply a term used to describe an algorithm that allows recursive estimation of unobserved, time varying parameters, or variables in the system. Ordinarily, it is different from forecasting in that forecasts could be made far into the future; whereas filtering obtains estimates of "unobservables" for the same time period as the information set (i.e. it allows extrapolation for regular albeit futuristic time lags). The Kalman filter, first developed by Kalman (1960), is a discrete, recursive linear filter, and was developed for use in aerospace engineering applications; it was used in the Apollo program in 1960 (Grewal and Andrews, 2010) and subsequently adopted by statisticians and econometricians. The basic idea behind the filter is simple - to arrive at a conditional density function of the unobservables using Bayes' theorem, one needs the functional form of relationship with observables, an equation of motion and assumptions regarding the distribution of error terms. The filter uses the current observation to predict the next period's value of unobservable and then uses the realisation next period to update that forecast. The Linear Kalman filter is optimal (i.e. with minimum Mean Squared Error (MSE)) if the observed variable and the noise are jointly Gaussian. The use of Kalman filtering in estimation and modelling by researchers is on the increase in the last couple of decades; noteworthy are the works of Bocquet and Sakov (2012), Bocquet (2011) and Hunt *et al* (2007). The structTS is a "stats" function used for fitting basic structural time series models (Harvey, 1989) to univariate variables. It is quite reliable and fast, and is hence used as a benchmark in comparisons of Univariate Time Series (UTS) models; it is the model to "beat" if any predictor happens to be a candidate for univariate prediction of "latent" variables.

Structural Model for a Time Series (structTS)

According to Harvey (1989), a structural time series model is one which is set up in terms of components which have a direct interpretation. A univariate structural model is not intended to represent the underlying data generation process. Rather it aims to present the "stylised facts" of a series in terms of a decomposition into components such as trends, seasonal and cycle. These quantities are of interest in themselves. Furthermore, they highlight the feature of a series which must be accounted for by a properly formulated behavioural model. Prediction from a univariate model is naïve in the sense that it is just an extrapolation of past movements. Nevertheless, it is often quite effective and it provides a yardstick against which the performance of more elaborate models may be assessed. Usually, any attempts to capture a data generation process will "amount" to, using a sample of the observed data to derive a model which will, at least, "perform" equally well as the StructTS, in the sense that it also captures the features of the data generation process, at least, as much as StructTS does. Also the derived model, should enable us to; estimate parameters, using any statistical technique (e.g. method of maximum likelihood), make predictions and construct confidence intervals for forecasts made from it, derive its refinement or generalization (i.e. if it exists), carry-out statistical tests to check its "adequacy" for the data generation process.

MATERIALS AND METHODS

The Algorithm for Continuous Forecasting of Inflation Rates through Kalman Filtering

With the assumption that the state of the monthly inflation rate (MIR) at a time *t* evolved from the former (i.e. last month's) rate that has the time (*t*-1) according to the model;

$$x_t = A_t x_{t-1} + B_t u_t + w_t$$

$$y_t = F_t x_t + v_t$$

Where; x_t is the state vector containing the terms of interest for the MIR system at time, *t*, u_t is the effect of a "unit" monthly attempts towards controlling the MIR, w_t contains the model errors and noises, per month, as such, it possesses a multivariate Gaussian distribution (with mean zero) and its pertinent covariance matrix can be denoted by G_t , A_t is the state transition matrix, at time t, "capturing" the transition that took place between the $(t-1)$ and t time periods, B_t is the control input matrix, since the attempts to control the MIR "comes" in various guises, the coefficients for the magnitude of each attempts are in this matrix. With respect to equation (2); y_t is the vector of n observations (i.e. measurements of MIR), F_t is the transformation matrix that maps the state vector parameters into the measurement domain (MIR), v_t is the vector containing the noise terms associated with each measurement in its vector. Like the process noise, the measurement noise is also assumed to be Gaussian distributed with zero mean and covariance matrix H_t.

The model dynamics for our discrete time MIR system is simplified by setting

$$B_t = 0 \quad \forall t$$

this simplification is borne out of our inability to identify and quantify all probable attempts at making y_t possess the barest minimal value, further equation (3) can also be interpreted to mean that all attempts to control MIR (i.e. y_t) towards a desirably "low" value is fruitless. However, with n observations, the system reduces to

$$x_t = A_t x_{t-1} + w_t$$

$$y_t = F_t x_t + v_t$$

With $A_t, F_t \in M_{n\times n}$ (i.e. n x n matrices), $y_t, x_t, x_{t-1}, w_t, v_t \in M_{n\times 1}$ (i.e. a column vector) and $\forall t$, $E(v_t) = 0$ and $\text{cov}(v_t) = H_t$. After the first transition, equation (2) remains as it is but equation (4) would "reflect" the transition and the system becomes

$$x_{t+1} = A_t x_t + w_{t+1}$$

$$y_t = F_t x_t + v_t$$

With $x_0 \to N(a_0, P_0)$, $w_t \to N(0, G_t)$ and $v_t \to N(0, H_t)$. By assuming that $B_t = \{y_i | 1 \le i \le t\}$ contains all the smooth and filtered observations, then we can estimate the best true state $\widehat{x}_t$ of x_t that minimizes the variance (or mean squared error) according as whether $\widehat{x}_t$ is unbiased (or biased) respectively, such that

$$E\left\{(x_t - \widehat{x}_t)^T (x_t - \widehat{x}_t)\right\} = trace\left[E\left\{(x_t - \widehat{x}_t)(x_t - \widehat{x}_t)^T\right\}\right] = trace\left\{\widehat{P}_t\right\}$$

Where the superscript T is for transpose. At the beginning (i.e. at time t = 0), the initial information is supplied and used to estimate x_0 and its allied variance. That is

$$\widehat{x}_0 = E(x_0) = a_0$$

$$\widehat{P}_0 = E\left\{(x_0 - \widehat{x}_0)(x_0 - \widehat{x}_0)^T\right\} = P_0$$

Armed with equation (7), we attempt to predict x_1 through

$$x_1^b = E\{x_1 | \hat{x}_0\} = E\{A_0 x_0 + w_1 | \hat{x}_0\} = A_0 x_0$$

The resulting error will be

$$e_i^b = x_1 - x_1^b = A_0 x_0 - A_0 \hat{x}_0 + w_1 = A_0 (x_0 - \hat{x}_0) + w_1 = A_0 \hat{e}_0 + w_1$$

Which leads us to the covariance

$$P_i^b = E\left\{e_i^b \left(e_i^b\right)^T\right\} = E\left\{\left(A_0 \hat{e}_0 + w_1\right)\left(A_0 \hat{e}_0 + w_1\right)^T\right\} = A_0 \hat{P}_0 A_0^T + G_1$$

We continue by predicting the next observation as follows;

$$E\left\{y_1 \left| x_1 = x_1^b \right.\right\} = E\left\{F_1 y_1 + v_1 \left| x_1 = x_1^b \right.\right\} = F_1 y_1^b$$

And

$$\text{cov}\left\{y_1 \left| x_1 = x_1^b \right.\right\} = E\left\{\left(y_1 - E\left[y_1 \left| x_1^b \right.\right]\right)\left(y_1 - E\left[y_1 \left| x_1^b \right.\right]\right)^T\right\} = E\left\{v_1 v_1^T\right\} = H_1$$

Continuing in this fashion, we later forcasted from (t – 1) to t as follows;

$$x_t^b = A_{t-1} \hat{x}_{t-1}, \quad e_t^b = x_t - x_t^b = A_{t-1}\left(x_t - \hat{x}_{t-1}\right) + w_t = A_{t-1} \hat{e}_{t-1} + w_t$$

Consequently;

$$P_t^b = E\left[e_t^b \left(e_t^b\right)^T\right] = E\left[\left(A_{t-1} \hat{e}_{t-1} + w_t\right)\left(A_{t-1} \hat{e}_{t-1} + w_t\right)^T\right] = A_{t-1} \hat{P}_{t-1} A_{t-1}^T + G_t$$

Hence the model for predicting our observations will be;

$$E\left\{y_t \left| x_t = x_t^b \right.\right\} = E\left[F_t x_t + v_t \left| x_t^b \right.\right] = F_t x_t^b$$

With

$$\text{cov}\left(y_t \left| x_t^b \right.\right) = \text{cov}\left(v_t\right) = H_t$$

The pseudo-Bayesian explanation of the estimated state variable, $\hat{x}_t$ (posterior), goes thus;

$$\hat{x}_t = x_t^b + \kappa_t \left\{y_t - F_t x_t^b\right\}$$

The resulting error will be

$$e_i^b = x_1 - x_1^b = A_0 x_0 - A_0 \hat{x}_0 + w_1 = A_0\left(x_0 - \hat{x}_0\right) + w_1 = A_0 \hat{e}_0 + w_1 \tag{10}$$

Which leads us to the covariance

$$P_i^b = E\left\{e_i^b\left(e_i^b\right)^T\right\} = E\left\{\left(A_0\hat{e}_0 + w_1\right)\left(A_0\hat{e}_0 + w_1\right)^T\right\} = A_0\hat{P}_0 A_0^T + G_1 \tag{11}$$

We continue by predicting the next observation as follows;

$$E\left\{y_1 \left| x_1 = x_1^b \right.\right\} = E\left\{F_1 y_1 + v_1 \left| x_1 = x_1^b \right.\right\} = F_1 y_1^b \tag{12}$$

And

$$\operatorname{cov}\left\{y_1 \left| x_1 = x_1^b \right.\right\} = E\left\{\left(y_1 - E\left[y_1 \left| x_1^b \right.\right]\right)\left(y_1 - E\left[y_1 \left| x_1^b \right.\right]\right)^T\right\} = E\left\{v_1 v_1^T\right\} = H_1 \tag{13}$$

Continuing in this fashion, we later forcasted from (t – 1) to t as follows;

$$x_t^b = A_{t-1}\hat{x}_{t-1}, \quad e_t^b = x_t - x_t^b = A_{t-1}\left(x_t - \hat{x}_{t-1}\right) + w_t = A_{t-1}\hat{e}_{t-1} + w_t \tag{14}$$

Consequently;

$$P_t^b = E\left[e_t^b\left(e_t^b\right)^T\right] = E\left[\left(A_{t-1}\hat{e}_{t-1} + w_t\right)\left(A_{t-1}\hat{e}_{t-1} + w_t\right)^T\right] = A_{t-1}\hat{P}_{t-1}A_{t-1}^T + G_t \tag{15}$$

Hence the model for predicting our observations will be;

$$E\left\{y_t \left| x_t = x_t^b \right.\right\} = E\left[F_t x_t + v_t \left| x_t^b \right.\right] = F_t x_t^b \tag{16}$$

With

$$\operatorname{cov}\left(y_t \left| x_t^b \right.\right) = \operatorname{cov}\left(v_t\right) = H_t \tag{17}$$

The pseudo-Bayesian explanation of the estimated state variable, $\hat{x}_t$ (posterior), goes thus;

$$\hat{x}_t = x_t^b + \kappa_t\left\{y_t - F_t x_t^b\right\} \tag{18}$$

Where the; prior is $x_t^b = A_{t-1}\hat{x}_{t-1}$, Kalman gain is κ_t , innovation is $y_t - F_t x_t^b = (F_t x_t + v_t) - F_t(A_{t-1}\hat{x}_{t-1})$ and $x_t = A_{t-1}x_{t-1} + w_t$. Consequently, the posterior and pertinent analysis are as follows:

$$\hat{x}_t = x_t^b + \kappa_t\left\{F_t\left(x_t - x_t^b\right) + v_t\right\} = A_{t-1}\hat{x}_{t-1} + \kappa_t\left\{F_t A_{t-1}\left(x_{t-1} - \hat{x}_{t-1}\right) + F_t w_t + v_t\right\} \tag{19}$$

$$\widehat{e}_t = x_t - \widehat{x}_t = A_{t-1}\widehat{e}_{t-1} - \kappa_t F_t A_{t-1}\widehat{e}_{t-1} + \left(\mathrm{I} - \kappa_t F_t\right) w_t - \kappa_t v_t = \left(\mathrm{I} - \kappa_t F_t\right)\left\{A_{t-1}\widehat{e}_{t-1} + w_t\right\} - \kappa_t v_t \tag{20}$$

$$\widehat{P}_t = E\left\{\widehat{e}_t\left(\widehat{e}_t\right)^T\right\} = \left(\mathrm{I} - \kappa_t F_t\right) E\left\{\left(A_{t-1}\widehat{e}_{t-1} + w_t\right)\left(A_{t-1}\widehat{e}_{t-1} + w_t\right)^T\right\} = \left(\mathrm{I} - \kappa_t F_t\right)^T + \kappa_t E\left(v_t v_t^T\right)\kappa_t^T \tag{21}$$

The pertinent covariance then becomes

$$\begin{aligned}\widehat{P}_t &= \left(\mathrm{I} - \kappa_t F_t\right)\left\{A_{t-1}\widehat{P}_{t-1}A_{t-1}^T + G_t\right\}\left(\mathrm{I} - \kappa_t F_t\right)^T + \kappa_t H_t \kappa_t^T = \left(\mathrm{I} - \kappa_t F_t\right) P_t^b \left(\mathrm{I} - \kappa_t F_t\right)^T + \kappa_t F_t \kappa_t^T \\ &= P_t^b - \kappa_t F_t P_t^b - P_t^b F_t^T \kappa_t^T + \kappa_t M_t \kappa_t^T\end{aligned} \tag{22}$$

Where $M_t = \left\{F_t P_t^b F_t^T + H_t\right\}$.

The following conditions, through the Kalman gain (κ_t), will enable us to minimize the total variance:

1. Let us start by first stating that (Faragher, 2012); $\kappa_t = P_t^b F_t^T M_t^{-1} = \dfrac{P_t^b F_t^T}{\left(F_t P_t^b F_t^T + H_t\right)}$ 23)
2. As shown in Harvey and Pierse (2012) by putting equation (23) into (22) we have;
   $$\widehat{P}_t = P_t^b - \kappa_t F_t P_t^b = \left(\mathrm{I} - \kappa_t F_t\right) P_t^b \tag{24}$$

The interpretation of Kalman gain (κ_t) when $I = F_t \in M_{n \times n}$,

This implies that by equation (18);

$$\widehat{x}_t = x_t^b + \kappa_t\left\{y - F x_t^b\right\} = x_t^b + \kappa_t\left\{y - x_t^b\right\} = \left(I - \kappa_t\right) x_t^b + \kappa_t y \tag{25}$$

And hence;

$$\widehat{x}_{jt} = \left(\frac{H_{jj}}{P_{jj}^b + H_{jj}}\right) x_{jt}^b + \left(\frac{P_{jj}^b}{P_{jj}^b + H_{jj}}\right) y_{jt} \tag{26}$$

Consequently; If P_{jj}^{b} is large, y_{jt} has a larger weight, in comparison with x_{jt}^{b}.
The algorithm will take one through the following stages:
1. Build the pertinent model.
2. Start-up the process by initializing the appropriate 'variables'.
3. Start iterating using the 'observed' as impetus as you go on.

RESULTS AND DISCUSSION

Illustrative Example

Here, we shall illustrate, using the data on the Nigerian economy, the monthly inflation rates of the economy between 2011 and 2015 are plotted in figure 1 below.

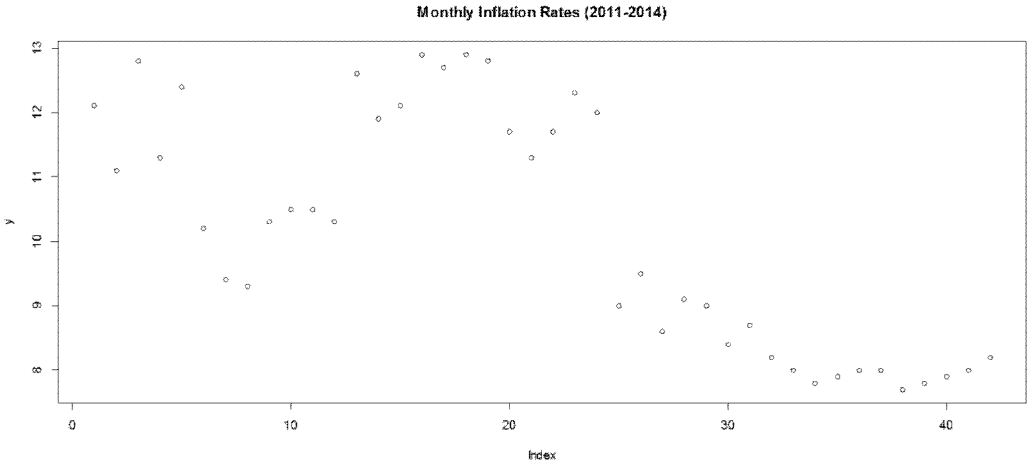

Figure 1: Showing the ordinary plot of the monthly inflation rates from January, 2011 to June, 2014 (Source: Central Bank of Nigeria (CBN) "FULL REPORT", 2014).

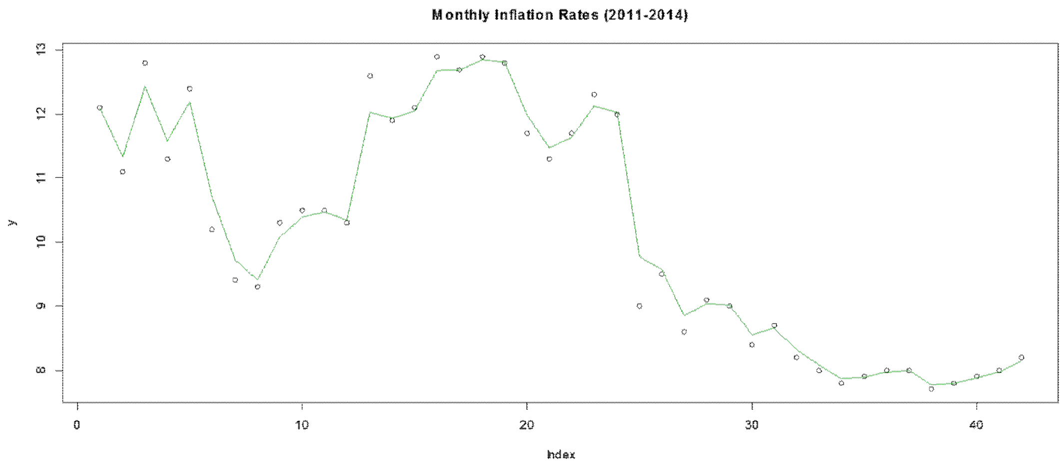

Figure 2: Showing the plot of the structural time series (structTS) model for the monthly inflation rates.

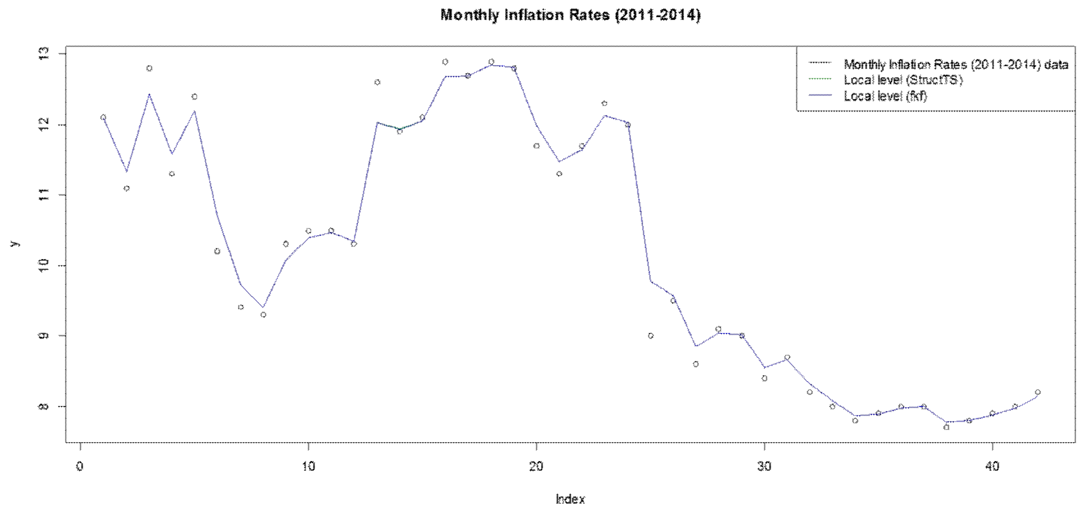

Figure 3: Showing the; "dots" plot of the observed monthly inflation rates, structural time series model plot and the Kalman filtering prediction.

Note that the Kalman filtering predictions plot superimposes itself over the structural time series model plot. Which means that it predicts the monthly inflation rates "efficiently".

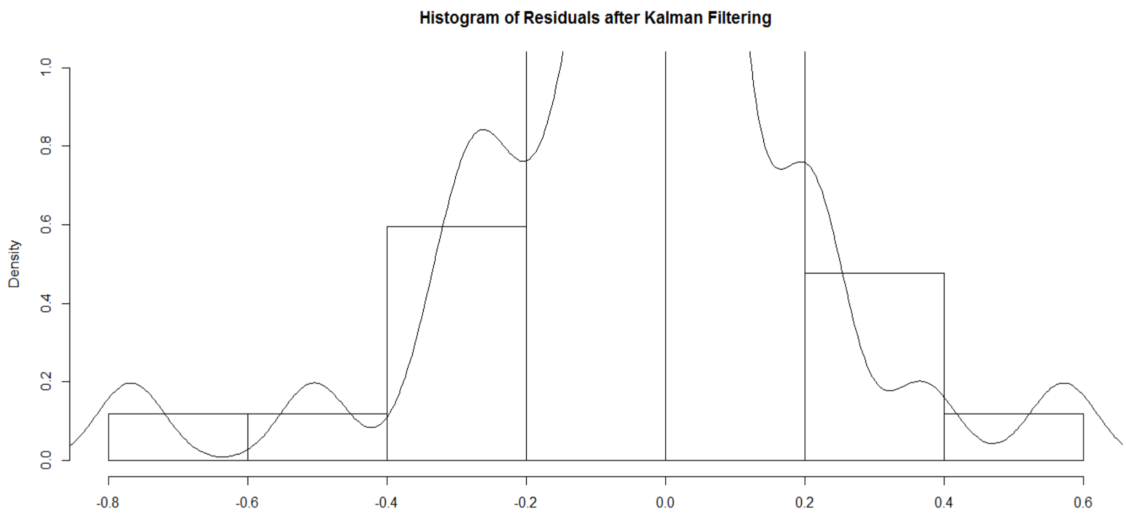

Figure 4: Showing the normality of the residuals obtained when Kalman Filtering was used to predict the monthly inflation rates.

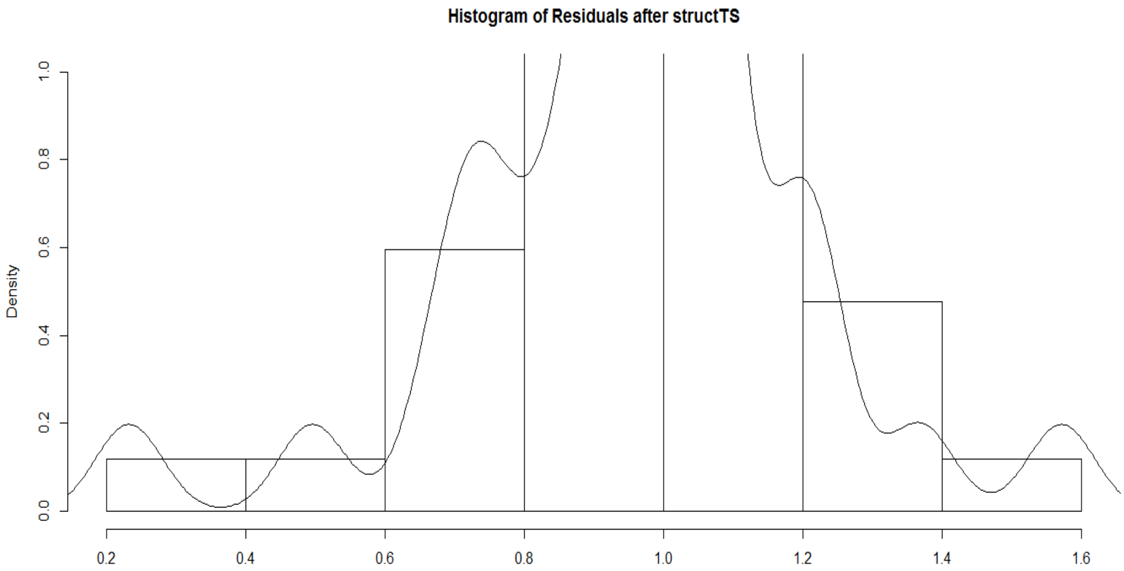

Figure 5: The normality of the residuals after structTS has been used as predictor for the observed monthly inflation rates.

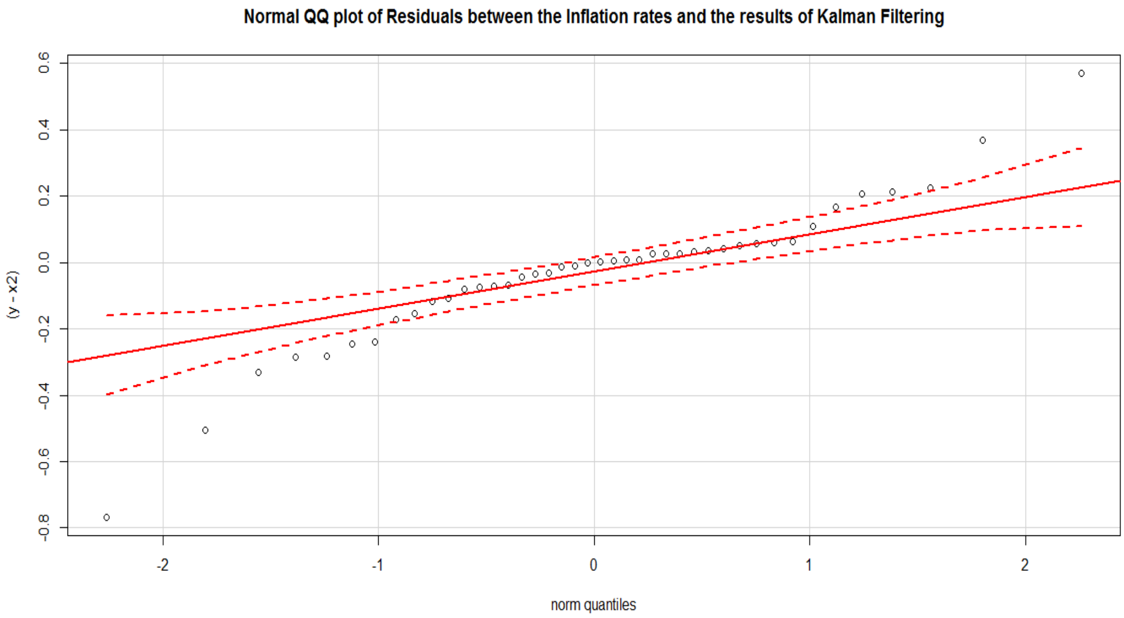

Figure 6: Showing the normal qq() plot of the residuals when Kalman Filtering was used to predict the observed monthly inflation rates.

Figure 7: Showing the normal qq() plot of the residuals when the structTS was used to predict the observed monthly inflation rates.

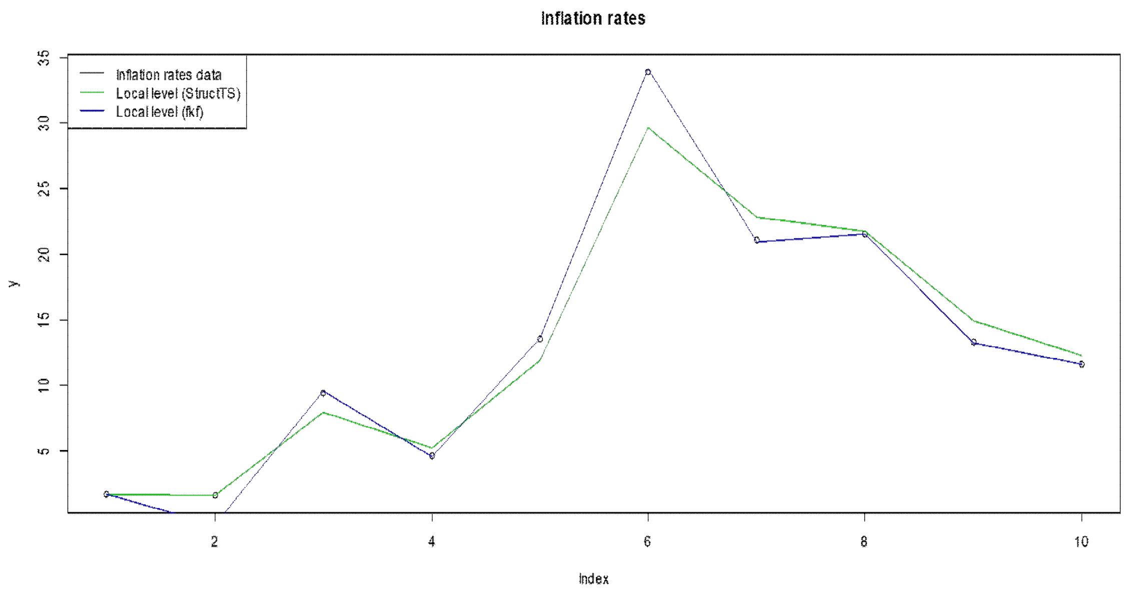

Figure 8: The superimposing of the StructTS and Kalman filtering plots for easy comparison of their performances.

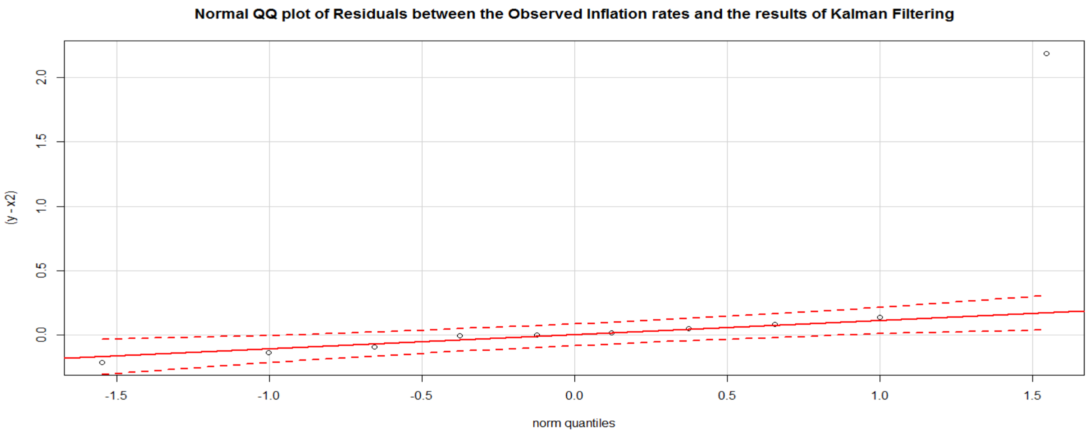

Figure 9: The normal QQ plot of Residuals between the observed inflation rates and the results of Kalman filtering showing how close to "perfection" the fit is.

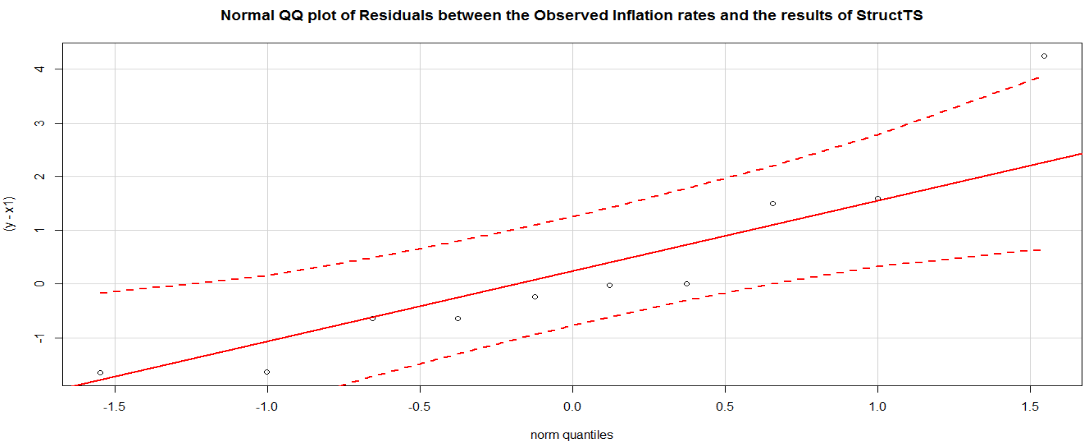

Figure 10: The normal QQ plot of Residuals between the observed inflation rates and the results of StructTS.

DISCUSSION AND CONCLUSION

Figure 1 contains ordinary (dot) plot of the monthly inflation rates from January, 2011 to June, 2014, the performance of the structural time series plot, for the data, is contained in Figure 2. The figure 3 shows that the performance of Kalman filtering (fkf) equals that of the structural time series, hence the diagrammatic representation (Figure 3) ends up in a superimposition. Figures 4 and 5 further show that the respective residuals are both normally distributed, but whilst the mean for the Kalman fitering model is zero (0) that of the structural time series model has the mean at 1.0 which gives an incline that the Kalman Filtering model gives a better fit the structural time series model globally, this fact is adequately buttressed by the qq-plots of figures 6 and 7. In order to show a numerical illustration, another data from the source of the plot in figure 1 was taken and by superimposing the performance plots of both models, figure 8 results which shows that the Kalman filtering model fits the data better than the structural time series model. The respective qq-plots of figures 9 and 10 collectively show that the Kalman filtering model is better. In order to complete this diagnostics and offer numerical evidences, the following set of codes give the channel through which the respective Akaike Information Criteria (AIC) values for the two models can be obtained. That is for the structural time series model, by giving the command;

```
> library(car)
> x1<-fitted(fit.stats)
> x2<-ts(fkf.obj$att[1, ], start = start(y), frequency = frequency(y))
> lm_fit<-lm(y~x1)
> summary(lm_fit)
```

To R (preferably version 3.1.1), the result obtained was;

```
Call:
lm(formula = y ~ x1)

Residuals:
    Min        1Q       Median       3Q        Max
-2.46529  -0.96163  -0.03377    1.15237   3.04282
Coefficients:
              Estimate  Std. Error  t value  Pr(>|t|)
(Intercept)  -0.4907     1.0030      -0.489   0.638
x1            1.0573     0.0637      16.599   1.75e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 1.799 on 8 degrees of freedom
Multiple R-squared:  0.9718,    Adjusted R-squared:  0.9683
F-statistic: 275.5 on 1 and 8 DF,  p-value: 1.753e-07
```

Since the closer the adjusted R-squared (i.e. 0.9683) to 1, the better the fit of the model to the data and also the smaller the p-value (i.e. 1.753e-07) the more reliable the summary (i.e. test), then this result implies that the structural time series model can be used to model economic inflation data and that this result is reliable. Now, if the command pertaining to the Kalman filtering model is given, that is by merely using x2 in-place of x1 in the code; lm_fit<-lm(y~x1), the result becomes;

```
Call:

lm(formula = y ~ x2)
Residuals:
   Min       1Q        Median      3Q        Max
-0.5868   -0.3983   -0.1727    0.1715    1.5226
Coefficients:
```

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	0.64447	0.33782	1.908	0.0929	.
x2	0.96609	0.02066	46.764	4.83e-11	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.6466 on 8 degrees of freedom
Multiple R-squared: 0.9964, Adjusted R-squared: 0.9959
F-statistic: 2187 on 1 and 8 DF, p-value: 4.833e-11

This result also shows that the Kalman filtering model is better than the structural time series model, considering the fact that the adjusted R-squared (i.e. 0.9959) is closer to 1 than the adjusted R-squared for the structural time series model. Also, the p-value here (i.e 4.833e-11) is smaller than the one for the structural time series model.
A more convincing numerical evidence is the calculation of their AIC, which for the Kalman filtering model will be obtained by giving the command; AIC(lm_fit), to R, to obtain the result; 23.42674. By repeating the same process for the structural time series model, the result is 43.89233. And since the model with the lower AIC is better than the other model for the data, then the Kalman filtering model is obviously better than the structural time series model for fitting economic inflation data.

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