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HIERARCHICAL MODELLING OF INDOOR AND OUTDOOR (RESIDENTIAL) RADON DATA (RRD)

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ABSTRACT

This work, proposes a Hierarchical Modelling (HM) for the indoor and outdoor Residential Radon Data (RRD). Indoor RRD and outdoor RRD are seen as distinct "hierarchies" of carcinogenic radioactive radon and both hierarchies constitute the least exposure that can be experienced by an individual. Works on this issue have always been based on complicated models, even for single instances of both indoor and outdoor residential radon. Our proposed method can be used to analyse effectively the many-to-many (it, however, becomes numerically clumsy if more than 5-to-5 instances are considered) instances of residential radon, although we have illustrated, here, using a three-to-three situation. Our preference of this method is based on its simplicity, and probable higher precision, as compared with the complexity involved in other methods on the same issue. The data used for the illustration of our models were taken from the indoors (i.e. living-room, bedroom and the kitchen) and the rest outdoors (i.e. verandah, car-park and the well-water shed) of a residential building in a lightly populated estate (i.e. Asero housing estate). Observations were taken on a daily basis throughout the dry season covering ninety days (i.e. January, February and March), this constitutes our season I (i.e. dry). The same was repeated in the season II (i.e. wet) which was taken at the beginning of June through July and August.

Key words: HM, RRD, Ordinary Least Squares (OLS), Pseudo-clusters and Pseudo-strata, R packages.

INTRODUCTION

Enough has been said about potential risks and health hazards associated with elevated levels of residential radon (Al Zabadil et al, 2012; Chege et al, 2009; Darby et al, 2001; Fitzpatrick-Lewis et al, 2010; Nero et al, 1994; Price, 1995; Singh, 2010; Smith and Oleson, 2008). The need to assess the least and maximum exposures, a hypothetical member of a community can experience,

cannot be emphasized because the inhabitants of such a community will always desire to know how "friendly" their environment is with respect to "freely available", carcinogenic radioactive radon, more so, if they share their neighbourhoods with rocks, and quarries, Electricity and nuclear power stations (the Chernobyl and Fukushima are two notable accidents that victims cannot forget easily). Even when there are no accidents,

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inhabitants of "rocky" environments such as those obtainable in Abeokuta, Ogun state, Nigeria have enough to worry about concerning carcinogenic radon. Concerning HM, there are locations; within a household (e.g. sitting (living) room, kitchen, toilets and bedroom) termed as indoor areas, between households (e.g. children play areas and immediate environments or compounds, verandas and car parking areas) termed as outdoor areas. These two categories of areas are identified as "hierarchies" or levels and to each of them pertinent models are fitted; In the aggregate of the two hierarchies are models upon which the statistical analyses are based. The fact that factors involved with RRD (i.e. Temperature, Pressure, Relative-Humidity etc.) are all quantifiable makes it seem natural for us to expect the radon emission to be a function of the levels of these variables also. Hence, either way, we can say that "there are hierarchies among the physical components of residential radon as well as the locality within which it is found".

DATA COLLECTION

Here, our equipment (i.e. radon emission detector is called Radon Scout TM, it is made by SARAD GmBh, Germany) consist of six distinct units three of which were placed indoors (i.e. living-room, bedroom and the kitchen) and the rest outdoors (i.e. verandah, car-park and the well-water shed). Observations were taken on a daily basis throughout the dry season covering ninety days (i.e. January, February and March), this constitutes our season I (i.e. dry). The same was repeated in the season II (i.e. wet) which was taken at the beginning of June through July and August. Radon Scout measures radon concentration in Bq m-3, as well as Temperature in \circ C, Relative humidity in percentage of concentration etc. But

this present work is based on the measured radon concentration alone.

METHODOLOGY

A methodology of HM, entails that we write two models (Wright and London, 2009), each of which will capture the scenario at each of the two identified hierarchies within a hypothetical residence (i.e. indoor and outdoor). Now, let us first assume, for the sake of brevity that there is one instance each of both hierarchies (i.e. indoor and outdoor). That is predictor y for indoor and v for outdoor.

Let us first consider regression models in which only intercepts vary (i.e. equal slopes). use the index, j, for a hypothetical indoor radon measurement and k, for the outdoor radon measurement associated with the indoor j. Then, we have;

$$
x_j = \alpha_k + \beta y_j + \varepsilon_j, j = 1, 2, \dots, m \quad (indoor)
$$

\n
$$
\alpha_k = a + b v_k + \xi_k, k = 1, 2, \dots, K \quad (outdoor)
$$
\n(3.1)

Where $\int y_j$ and $\int y_k$ are estimators, $\int z_j$ and ε

 $^{\mathcal{E}_{k}}$ are independent residual quantities, at the indoor and outdoor radon hierarchies respectively.

However both the intercept and slope can be allowed to vary but the work will become cumbersome (Gelman and Hill, 2007); the pertinent model for this situation is as stated below (equation 3.2):

$$
x_j = \alpha_{k(j)} + \beta y_j + \varepsilon_j, j = 1, 2, ..., m \text{ (indoor)}
$$

\n
$$
\alpha_k = a_0 + b_0 y_k + \xi_{k1}, k = 1, 2, ..., K \text{ (outdoor)}
$$

\n
$$
\beta_k = a_1 + b_1 y_k + \xi_{k2}, k = 1, 2, ..., K \text{ (outdoor)}
$$
\n(3.2)

However, use HM whenever your data is grouped (or nested) in more than one category (for example, states, countries, etc). HM will allow its user to; study the effects that vary entity by entity (or group-bygroup), and estimate group level averages, now, this is a vital advantage because regular regression ignores the average variation between entities. Besides, individual regression may face sample problems and lack of generalization.

Further discussion on these two models, concerning radon, can be found in Gelman (2005), Gelman (2006), Gelman and Hill (2007) and Tranmer and Elliot (2007). Pioneering works of HM can be found in

Goldstein (1986), Goldstein and McDonald (1988) and Goldstein (1991). A series of approaches can be found in works such as; Bates (2005), Hedeker (2007), Bolker et al (2008), Skrondal and Rabe-Hesbeth (2009), Wright and London (2009), Kenny and Hoyt (2009), Nikita (2010), Grilli and Rampichini (2012), and Schliep and Hoeting (2013).

Our proposed method assumes that each multilevel is actually a "poly-level" that can be split into two or more levels (according to similarity, convenience and the number of instances involved altogether) or compartments in such a way that data entries in the same level are as similar as possible (with respect to the characteristics under study). It sustains but pays little attention to spacial proximity as a way of trading it off for higher "precision". To make things clearer, our work purports to carry-out analysis on the statistical states of the radon emission concentration of just adequately populated residential buildings in a social elites area (i.e. Asero housing estate) of Ogun state to enable our audience to infer

from it the quantity of radon concentration the inhabitants of Ogun state, in general are exposed to on a daily basis. Towards this end we start by saying let us see the entire analysis as a multilevel involving two levels (i.e. indoor and outdoor) which will bring to relevance either of the two models (i.e. (3.1) or (3.2) above), but rather since the demarcation into "indoor" and "outdoor" is not "too" vital to our analysis (or, at least, not as important as having a high precision measure of the said radon emission concentration), we "trade" it off for higher precision and see the problem as a HM with six hierarchies (each representing one of the six locations where our radon scout equipment were kept). Albeit our data entry procedure will not be devoid of the original classification into two (indoor and outdoor) areas but what is more important is the fact that the data entries within a hierarchy (i.e. indoor or outdoor) are as similar as possible. Hence the two hierarchies are seen as "pseudoclusters" within each of which are "pseudostrata" (the three distinct locations).

Our statistical analysis recognizes the fact that, and indeed assumes that, observations from distinct pseudo-strata in the same pseudo-cluster may be correlated. In the mathematical analysis used to examine the effects of this correlation, we let i refer to the day (i.e. replication), j to the pseudocluster and k to the pseudo-stratum; such that, for pseudo-strata in the same pseudocluster, subscripts i and j will be the same per day. We assume that there exists a

correlation \overline{P} between the experimental

errors $\frac{\varepsilon_{iju}}{i}$ and $\frac{\varepsilon_{ijv}}{i}$ for any pair of distinct pseudo-strata in the same pseudo-cluster. However, pseudo-strata in distinct pseudoclusters may be uncorrelated. Thus, we have

$$
E\left(\varepsilon_{iju}\varepsilon_{sv}\right) = \begin{cases} \rho\sigma^2, & \text{if } \left((i=s)\wedge(j=t)\right) \\ 0, & \text{if } \left((i\neq s)\vee(j\neq t)\right) \end{cases}
$$
 (3.3)

It is also possible that we have, in each pseudo-stratum two or more entries. Although there are two seasons involved with our RRD (i.e. wet and dry), but we may have reasons to take radon readings during the day and night. However, with two readings per pseudo-stratum, the error variance of a pseudo-cluster total becomes;

$$
H(\varepsilon_{\rm M}+\varepsilon_{\rm M})^2 = H(\varepsilon_{\rm M})^2 + H(\varepsilon_{\rm M})^2 + 2H(\varepsilon_{\rm M}\varepsilon_{\rm M}) = 2\sigma^2 + 2\sigma^2 = 2\sigma^2(1+\rho)
$$
\n(3.4)

The factor 2 is regarded as representing the effects of adding over two readings in each pseudo-stratum. Hence the error variance

per pseudo-stratum is
$$
\sigma^2(1+\rho)
$$
. With m
pseudo-strata per pseudo-cluster, the corre-

 $\sigma^2(1 + (m-1)\rho)$

sponding error variance is On the other hand, the error variance from the difference in the two readings in a pseudo-stratum gives;

$$
E\left(\varepsilon_{st1} - \varepsilon_{st2}\right)^2 = 2\sigma^2 \left(1 - \rho\right) \tag{3.5}
$$

 (3.3) Equation (3.5) is also the effective variance per pseudo-stratum and is invariant, irrespective of the number of readings inside a

pseudo-stratum. With our RRD , P^2 (i.e. correlation coefficient is always positive). As explained earlier in this work, the main effects of the pseudo-clusters (which will be referred to as A, for brevity) are less precisely estimated, as a trade-off for that of the pseudo-strata (which will be B, for brevity) and consequently, of the AB interaction. $\rho > 0$

As for the analysis of variance, we first compute the six pseudo-strata totals. Their sum of squares of deviations is partitioned in the usual way into 1 d.f. (i.e. degree of freedom) for replication, 1 for the main effects of A, and 1 for the experimental error applicable to a whole pseudo-cluster. All computations are divided by 2 to convert them to a pseudo -stratum basis. The six differences provide 2 d.f. which represents the main effect of B, 2 d.f. which represent the AB interactions, and the remaining 4 d.f. whose mean square gives an unbiased estimate of the pseudo-

strata error variance \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} All sums of squares are again divided by 2. The complete separation of degrees of freedom is shown in the table 1 below: $\sigma^2(1-\rho)$

Table 1: Analysis of variance for experiments where pseudo-strata are within pseudo-clusters.

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If the pseudo-clusters are in α locations and the pseudo-strata are in \sim locations, the subdivision of degrees of freedom in β

the analysis of variance is shown in table 2 below where pseudo-clusters are arranged in randomized blocks (r replicates):

Table 2: Partition of degrees of freedom for pseudo-strata within pseudo-cluster experiment.

Let \mathcal{E}_A and \mathcal{E}_B be the error mean squares for error (A) and error (B) respectively, on a pseudo-stratum basis. For the means, also

expressed on a pseudo-stratum basis, the standard errors shown in the table 3 below are pertinent:

Table 3: **Standard errors for the pseudo-strata in pseudo-cluster experiment. Comparisons**

The last comparison in table 3 contains both the main effect of pseudo-cluster and the (pseudo-cluster)(pseudo-stratum) interaction; consequently the appropriate error is

a weighted mean of $\left. \begin{array}{cc} \varepsilon_{\scriptscriptstyle A} & \text{and} \end{array} \right.$ $\left. \begin{array}{cc} \varepsilon_{\scriptscriptstyle B} & \text{.} \end{array} \right.$ This error also applies to the difference between two pseudo-clusters means which have different levels of pseudo-strata. Now, let t_A , t_B be the significant levels of "t" corresponding to the degrees of free-

dom in $\left. \frac{\varepsilon_{\scriptscriptstyle{A}}}{\varepsilon_{\scriptscriptstyle{B}}} \right.$ and $\left. \frac{\varepsilon_{\scriptscriptstyle{B}}}{\varepsilon_{\scriptscriptstyle{B}}} \right.$, respectively. The significance level of "t" is;

$$
t = \frac{(\beta - 1)\varepsilon_{B}t_{B} + \varepsilon_{A}t_{A}}{(\beta - 1)\varepsilon_{B} + \varepsilon_{A}}
$$
(3.6).

RESULTS

Table 4 contains an extract from our RRD, with indoor observations denoting indoor RRD (with its three locations observations

(i.e. In1, In2 and In3), and outdoor observations denoting outdoor RRD (with its three locations observations (i.e. Out1, Out2 and Out3). The observations spans through approximately three months (duration per season) and entries are recorded on a daily basis, the unit of measurements is "Bq/m3". Without loss of generality, we assume that our RRD can still be treated as randomized blocks scheme with 90 replications.

It is customary to compute the analysis of variance on a pseudo-strata basis. For avoidance of confusion, this will be stated clearly in the analysis of variance table itself.

The calculations involved can be presented in the following three steps.

Step I: Obtain the pseudo-cluster totals by the method appropriate to the design in which they are arranged.

Table 4: An extract of our RRD showing the way it was constructed from the in doors and outdoors observations.

Step II: This concerns the pseudo-strata. Their main effects are obtained directly.

Locations (i.e. indoor and outdoor):

 $(14617)^{2} + (10470)^{2} + ... + (12136)^{2}$ (3.12) $132989.63 = 5101844$ (3.12) 180 $+(10470)^2+...+($ $-132989.63=5$

The sum of squares for interactions between pseudo-strata and pseudo-clusters is found by subtraction. First calculate the total sum of squares for the two-way table that shows both sets.

$$
\frac{(8206)^2 + (6242)^2 + ... + (5665)^2}{90} - 132989.63 = 5289408
$$
 (3.13)

Total Radon:

Now,

Seasons X Locations: 5289408 – 5101844 – 33089.337 = 154475 (3.14)

Step III: Compute the total sum of squares among all pseudo-strata.

Total sum of squares:
$$
(95)^2 + (109)^2 + ... + (32)^2 - 132989.63 = 13849608
$$
 (3.15)

The sum of squares for error (B) is then found by subtraction in the analysis of variance table 5.

Table 5: **Analysis of variance on a pseudo-stratum basis for the residential radon emission experiment**

Here, all except replications and deviations are significant, at 99% confidence. In table 6, below are shown the means, with the

principal standard errors as obtained from table 2.

Table 6: Radon Concentration Means (Bq/m³

Standard error of difference between

Two season means:
$$
\sqrt{\frac{2(622.58)}{540}} = 1.52
$$
 (89 d.f.) (3.16)

Two location means:
$$
\sqrt{\frac{2(9506)}{180}} = 10.3
$$
 (890 d.f.) (3.17)

Two location means for one season:

$$
\sqrt{\frac{2(9506)}{90}} = 14.5
$$
 (890 d.f.) (3.18)

Two season means for a given location:
$$
\sqrt{\frac{2\{5(9506) + 622.58\}}{540}} = 13.4
$$
 (3.19)

DISCUSSION

The 5% levels of t are 1.98 and 1.96 respectively, for 89 and 890 degrees of freedom. Consequently, the 5% level for the last standard error (3.19) above is

 $5(9506)(1.96) + (622.58)(1.98)$ $5(9506)+(622.58)$

 $^{+}$

There is a remarkable difference between indoor and outdoor radon emission per season, a mere perusal of summaries and correlation tables for our RRD confirms this fact ahead of the more convincing proof provided by our analysis of variance table 5.

Table 7: **Summary of indoor and outdoor (RRD) season I**

 $=$

1.96

(3.20)

Table 8: **Summary of indoor and outdoor (RRD) season II**

Table 9: **Showing the correlation between indoor and outdoor entries for season I**

Table 10: **Showing the correlation between indoor and outdoor entries for season II**

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With respect to table 5, season as a factor is highly significant with the F_{cal} value 53.15 whilst the corresponding F_{tab} entry is just 6.63 (at 1% level of significance), this statistically shows that radon emission (in particular, our RRD) is usually affected by weather fluctuations. Locations (i.e. radon emission indoor readings versus the outdoor equivalents) as well as the interaction between seasons and locations are also both highly significant. These two statements confirm that radon emission is grossly affected by change in location and the interaction between change in location and seasonal fluctuations. The regression coefficient amounts to an increase of 0.1572 in the radon emission for each change in location. Deviation is not significant, though it seems lower than expectation.

CONCLUSION

The result presented in this paper has certainly broadened our intellects about residential radon emission around our homes but we still have a long way to go. We still have to look at what is happening at other less appropriate habitable areas (Alzabadil et al (2012) (e.g. down-town or ghetto Abeokuta) because our data for this work was taken from comparatively better habitable areas (i.e. Asero housing estate). Also radon emissions at some specific work places are of paramount importance (Dawodu et al (2011)) we need to know, among other things, the nature and state, with respect to safety, of radon emissions at the artisans' shops, classrooms and laboratories and offices to be able to determine how hazardous occupational radon emission can be. Our ultimate desire is to be able to construct radon emission maps of locations and hence of our nation Nigeria.

In Darby et al (2001), the individual and

ecological information concerning the exposure, of the inhabitants of south-west England, to residential radon was discussed as potential causes of lung cancer. The complication involved in this is apparent because the percentage contribution of each of the exposures, towards catching lung cancer, could not be estimated hence the authors concluded as follows "Findings suggest residential radon may increase COPD mortality. Further research is needed to confirm this finding and to better understand possible complex inter-relationships between radon, COPD and lung cancer."

In Nigeria, from inception till date, ours is the second attempt towards the quantification of the exposure to radon concentration of the inhabitants. The first was carried out by Ademola et al (2011), it was not on residential emission, it can only be an estimate of outdoor radon concentration because in it, the authors distributed and exposed seventy CR-39 tracks detectors in 35 high schools of the Oke-Ogun area (their study area) for three months and manually processed the detectors to determine the total number of tracks and finally estimated the radon concentration at 45 ± 27 Bq m⁻³ at the University Laboratory at Trieste, Italy.

Our results are more reliable than this single estimate, in the sense that, it contains; information about the significance of replications, seasons, locations, deviations and the combination of seasons and locations (all of which were found to be significant except deviations, as shown in table 5), two seasons means (dry and wet), six locations means (three indoors and three outdoors), the grand mean and standard errors of difference between two; season means, location means, location means for one season, season means for a given location (as shown in measures of spreads of RRD in season I (as shown in table 7) summary of measures of spreads of RRD in season II (as shown in table 8), correlation between indoor and outdoor entries for season I (as shown in table 9) and finally the correlation between indoor and outdoor entries for season II (as shown in table 10).

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